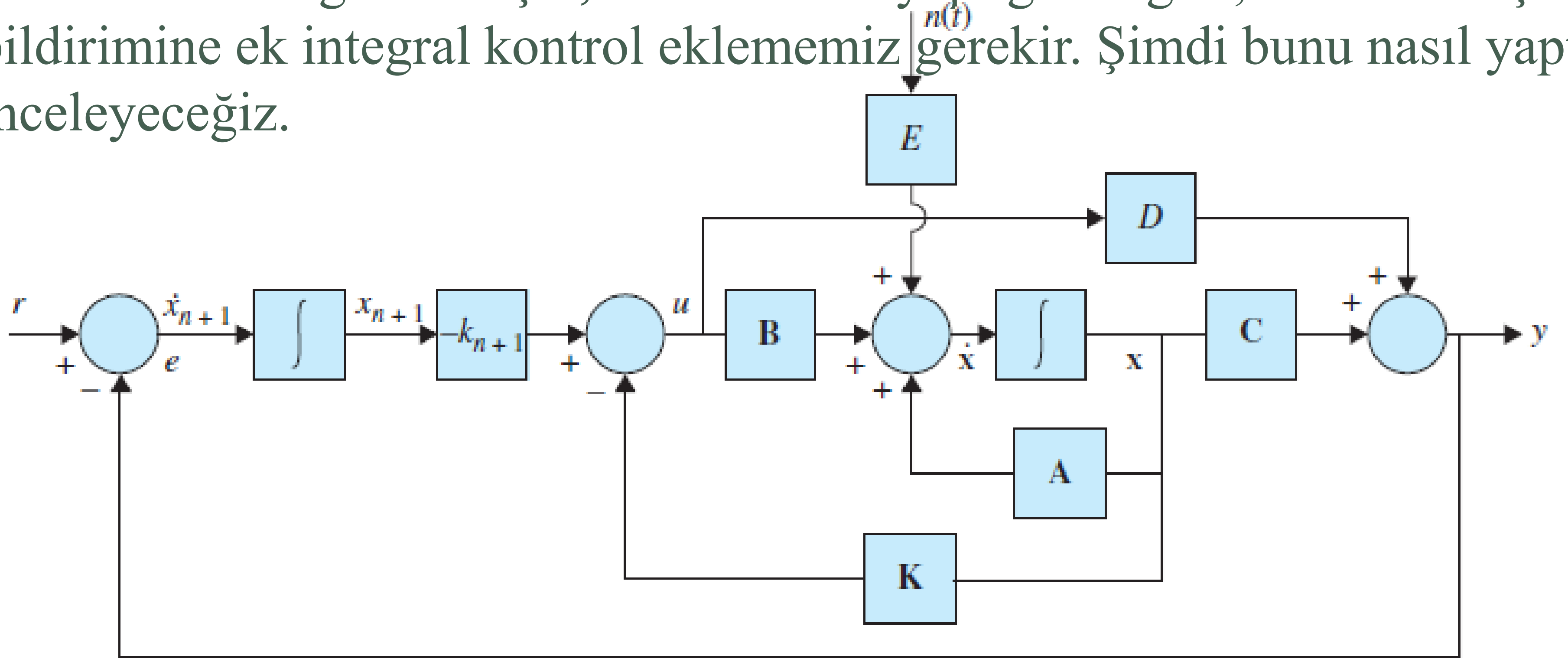
Modern Kontrol

Veren kişi

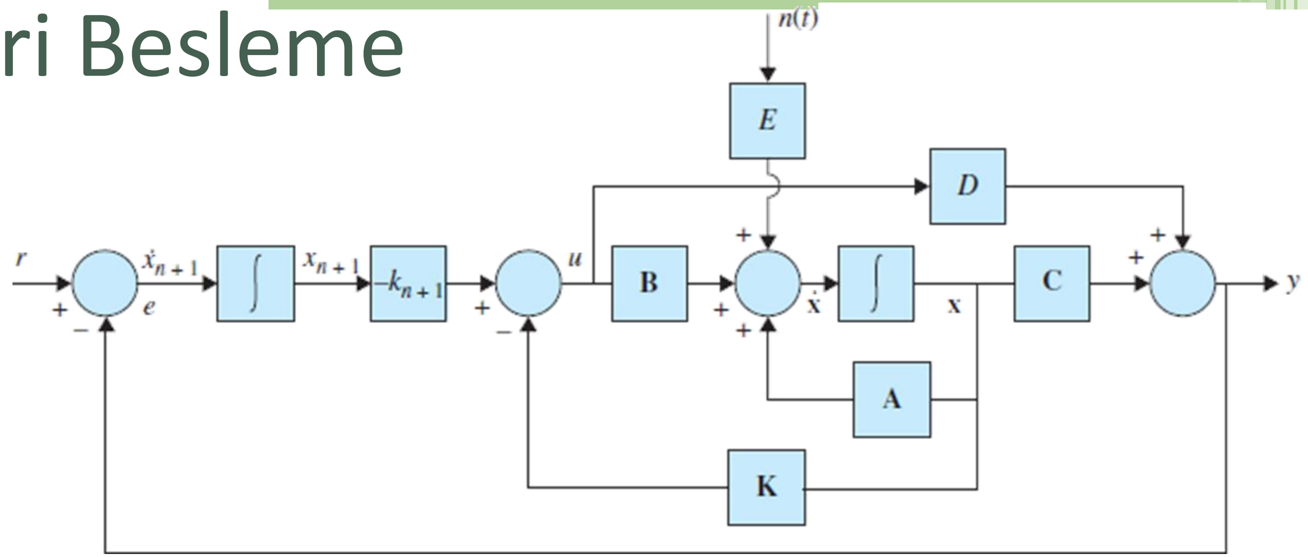
Dr. Öğr. Üyesi Nurdan Bilgin

İntegral Kontrollü Durum Geri Besleme

Şimdiye kadar çıkışın ve girişin sabit sayılar olduğu varsayımı ile problemleri çözdük; Bu varsayım regülatör sistemler için uygundur. Oysa mühendislerin uğraştığı problemlerin bir çoğu servo problemlerdir. Yani sistemin bir referansı takip etmesini isteriz. Bunu sağlamak için, PI kontrolde yaptığımız gibi, sabit kazanç durum geri bildirimine ek integral kontrol eklememiz gerekir. Şimdi bunu nasıl yaptığımızı inceleyeceğiz.



İntegral Kontrollü Durum Geri Besleme



Blok diyagramına bakacak olursak; girişteki integratörün sağ tarafı x_{n+1}olarak belirlenmiştir. Sistem dinamiği ise gürültü ifadesi ile beraber aşağıdaki şekildedir.

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t) + En(t)$$

Blok diyagramdan $\dot{x}_{n+1}$ ifadesinin referans giriş ile çıkış arasındaki fark olarak alındığını görebiliyoruz. Yani

$$\dot{x}_{n+1} = r(t) - y(t)$$

Çıkış ifadesi ise

$$y(t) = Cx(t) + Du(t)$$

Bu durumda kontrol giriş sinyali $u(t)$ sadece durum geri bildirimi ile ilişkili değil aynı zamanda integral geri bildirimle de ilişkili, Şöyleki

$$u(t) = -Kx(t) - k_{n+1}x_{n+1}(t)$$

Burada

$$K = [k_1\ k_2\ k_3\ \cdots\ k_n]$$

Gerçek kazanç elemanlarını tutan dizi matris, ve k_{n+1}integral geri bildirim kazancı.

İntegral Kontrollü Durum Geri Besleme

$$\dot{x}_{n+1} = r(t) - y(t)$$

$$-x_{n+1}(t) \triangleq \int_0^t y(\tau)d\tau$$

$$\Downarrow$$

$$\dot{x}_{n+1}(t) \triangleq -y(t) = -\underline{c}^T\underline{x} + r_k(t)$$

$$\begin{bmatrix} \underline{\dot{x}} \\ \dot{x}_{n+1} \end{bmatrix} = \begin{bmatrix} A & \vdots & \underline{0} \\ \cdots & \cdot & \cdots \\ -\underline{c}^T & \vdots & 0 \end{bmatrix} \begin{bmatrix} \underline{x} \\ \cdots \\ x_{n+1} \end{bmatrix} + \begin{bmatrix} \underline{b} \\ \cdots \\ 0 \end{bmatrix} u + \begin{bmatrix} \underline{0} \\ \cdots \\ 1 \end{bmatrix} r_k$$

$$\underline{\dot{x}}^* = \underline{A}^*\underline{x}^* + \underline{b}^*u + b_k^* r_k$$

$$y = -\underline{c}^{*T}\underline{x}^* + r_k$$

$$\underline{c}^{*T} = [-\underline{c}^T \vdots\, 0]$$

İntegral Kontrollü Durum Geri Besleme

$$\underline{\dot{x}}^* = \left(\underline{A}^* - \underline{b}^* k^{*T}\right)\underline{x}^* + b_k^* r_k$$

$$\left|\lambda I - \left(\underline{A}^* - \underline{b}^* k^{*T}\right)\right| = \prod_{i=1}^{n+1} (\lambda - \lambda_i^*)$$

Örnek

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 50 \\ -200 & -200 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 200 \end{bmatrix} u$$

$$\lambda_1 = \lambda_2 = -100$$

Yukarıda durum uzay formunda verilen ve özdeğerleri -100 olan sistem için durum geri bildirimli kontrolcü tasarlayın, ilk durum değişkeni x_1'in referans giriş r_1'i izlemesini sağlayın ve kontrollü sistemin tüm özdeğerlerinin -200'de olmasını sağlayın.

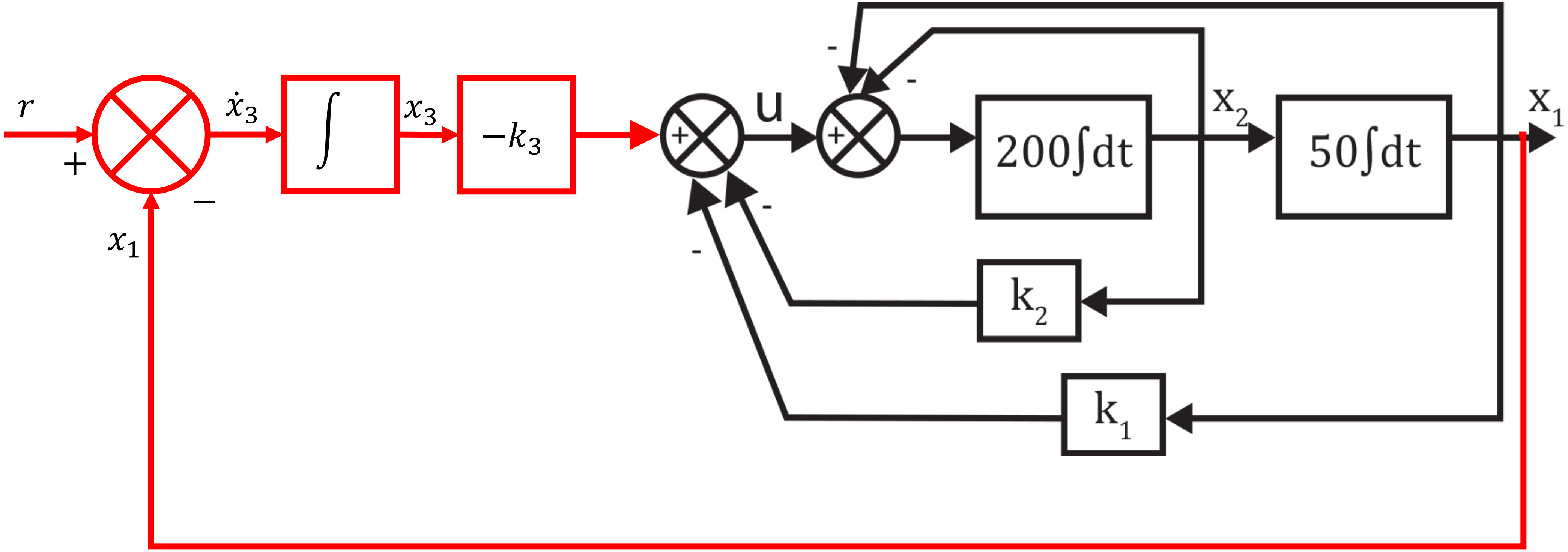
Çözüm;

$$A^* = \begin{bmatrix} 0 & 50 & \vdots & 0 \\ -200 & -200 & \vdots & 0 \\ \cdots & \cdots & \cdot & \cdots \\ -1 & 0 & \vdots & 0 \end{bmatrix}; b^* = \begin{bmatrix} 0 \\ 200 \\ \cdots \\ 0 \end{bmatrix}; b_1^* = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$u = -k_1 x_1 - k_2 x_2 - k_3 x_3$$

$$\left| \lambda I - \left(\underline{A}^* - \underline{b}^* {k^*}^T \right) \right| = (\lambda + 200)^3$$

$$k_1 = 11, \qquad k_2 = 2, \qquad k_3 = -800$$



r
+
−
x_1
$\dot{x}_3$
$\int$
x_3
$-k_3$
+
u
+
-
-
-
-
$200\int dt$
x_2
$50\int dt$
x_1
k_2
k_1

İntegral Kontrollü Durum Geri Besleme

Çok Girişli Sistemler

$$\underline{M} = \left[\left[\underbrace{\underline{b}_1 \vdots \underline{b}_2 \vdots \cdots \vdots \underline{b}_r}_{\underline{B}}\right] \vdots \underbrace{\underline{A}\underline{b}_1 \vdots \underline{A}\underline{b}_2 \vdots \cdots \vdots \underline{A}\underline{b}_r}_{\underline{A}\underline{B}} \vdots \cdots \vdots \left[\underline{A}^{n-1}\underline{b}_1 \vdots \underline{A}^{n-1}\underline{b}_2 \vdots \cdots \vdots \underline{A}^{n-1}\underline{b}_r\right]\right]$$

$$\Gamma = \left[\underline{b}_1 \vdots \underline{A}\underline{b}_1 \vdots \cdots \vdots \underline{A}^{\gamma_1 - 1}\underline{b}_1\right] \vdots \left[\underline{b}_2 \vdots \underline{A}\underline{b}_2 \vdots \cdots \vdots \underline{A}^{\gamma_2 - 1}\underline{b}_2\right] \vdots \cdots \vdots \left[\underline{b}_r \vdots \underline{A}\underline{b}_r \vdots \cdots \vdots \underline{A}^{\gamma_r - 1}\underline{b}_r\right]$$

$$\sum_{i=1}^{r} \gamma_i = \gamma_1 + \gamma_2 + \cdots \gamma_r = n$$

$$\gamma_i\!: i.\ giri\c{s}e\ ba\check{g}l\imath\ kontrol\ edilebilirlik\ indeksi$$

İntegral Kontrollü Durum Geri Besleme

diyelim ki $\underline{\alpha}_{ki}^{T}$, $\underline{\Gamma}^{-1}$ matrisinin k_i'inci satırı olsun. Burada $k_i = \sum_{i=1}^{i} \gamma_i \Rightarrow k_1 = \gamma_1; k_2 = \gamma_1 + \gamma_2$

$$\begin{bmatrix} \underline{\alpha}_{k1}^{T} \\ \underline{\alpha}_{k1}^{T} A \\ \underline{\alpha}_{k1}^{T} A^{\gamma_1 - 1} \\ \cdots \\ \underline{\alpha}_{k2}^{T} \\ \underline{\alpha}_{k2}^{T} A \\ \underline{\alpha}_{k2}^{T} A^{\gamma_2 - 1} \\ \cdots \end{bmatrix} = \underline{T}^{-1}$$

$\gamma_1\ satırları$

$\gamma_2\ satırları$

$n \times n$ Tekil olmayan matris

İntegral Kontrollü Durum Geri Besleme

$$\underline{x} = \underline{T}\,\underline{\hat{x}}$$

$$\underline{\dot{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u} \Rightarrow \underline{\dot{\hat{x}}} = \underline{\hat{A}}\,\underline{\hat{x}} + \underline{\hat{B}}\,\underline{u}$$

$$\underline{\hat{A}} = \underline{T}^{-1}\underline{A}\,\underline{T}, \qquad \underline{\hat{B}} = \underline{T}^{-1}\underline{B}$$

$$\underline{\hat{A}} = \begin{bmatrix} \underline{\hat{A}}_{11} & \underline{\hat{A}}_{12} & \cdots & \underline{\hat{A}}_{1r} \\ \underline{\hat{A}}_{21} & \underline{\hat{A}}_{22} & \cdots & \underline{\hat{A}}_{2r} \\ \vdots & \vdots & . & \vdots \\ \underline{\hat{A}}\_r_1 & \underline{\hat{A}}_{r2} & \cdots & \underline{\hat{A}}_{rr} \end{bmatrix}$$

$$\underline{\hat{A}}_{ii} = \begin{bmatrix} \ddots & 1 & 0 \\ 0 & \ddots & 1 \\ x.. & x.. & x.. \end{bmatrix}; \underline{\hat{A}}_{ij} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ x.. & x.. \end{bmatrix}$$

$$\underline{\hat{B}} = \begin{bmatrix} \underline{\hat{B}}_1 \\ \underline{\hat{B}}_2 \\ \vdots \\ \underline{\hat{B}}_r \end{bmatrix}; = \left(\underline{\hat{B}}_i\right)_{\gamma_i \times r} = \begin{bmatrix} & 0_{\gamma_{i-1} \times r} & \\ 0 & 1 & x \end{bmatrix}$$

$i.\,kolon$

İntegral Kontrollü Durum Geri Besleme

$$Q = \begin{bmatrix} q_1^T \\ q_2^T \\ \vdots \\ q_r^T \end{bmatrix}, \hat{B} = \bar{B}Q \Longrightarrow \bar{B}_i = \begin{bmatrix} & 0 & \\ 0 & 1 & 0 \end{bmatrix}$$

$i.kolon$

Örnek

$$\underline{A} = \begin{bmatrix} -2 & 1 & 2 \\ -1 & -2 & 2 \\ -2 & 0 & 2 \end{bmatrix}, \underline{B} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Özdeğerler$=-2, +j, -j; istenen$ $özdeğerler: -1, -2, -3$

$$|\underline{b}_1 \vdots \underline{A}\underline{b}_1 \vdots \underline{A^2}\underline{b}_1| = \begin{vmatrix} 0 & 1 & -4 \\ 1 & -2 & 3 \\ 0 & 0 & -2 \end{vmatrix} = 2$$

$$|\underline{b}_2 \vdots \underline{A}\underline{b}_2 \vdots \underline{A^2}\underline{b}_2| = \begin{vmatrix} 0 & 2 & 2 \\ 0 & 2 & -2 \\ 1 & 2 & 0 \end{vmatrix} = -8$$

Örnek devam

$$\underline{\Gamma} = \begin{bmatrix} 0 & 1 & \vdots & 0 \\ 1 & -2 & \vdots & 0 \\ 0 & 0 & \vdots & 1 \end{bmatrix} \; Seçilsin$$

$$\underline{\Gamma}^{-1} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$k_1 = \sum_{i=1}^{1} \gamma_i \Rightarrow k_1 = \gamma_1 = 2$$

$$k_2 = \sum_{i=1}^{2} \gamma_i \Rightarrow k_2 = 1\gamma_1 + 1\gamma_2$$

$$\gamma_1 = 2, \qquad \gamma_2 = 1, \qquad k_1 = \gamma_1 = 2, \qquad k_2 = \gamma_1 + \gamma_2 = 3$$

Örnek devam

$$T^{-1}=\begin{bmatrix}\begin{bmatrix}\underline{\gamma}_2^T\\ \underline{\gamma}_2^T A\end{bmatrix}\\ \cdots\\ \begin{bmatrix}\underline{\gamma}_3^T\end{bmatrix}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ -2 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}\Rightarrow T=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & -2\\ 0 & 0 & 1\end{bmatrix}$$

$$\underline{\hat{A}}=\underline{T}^{-1}\underline{A}\underline{T}=\begin{bmatrix}0 & 1 & 0\\ -9 & -4 & 10\\ -2 & 0 & 2\end{bmatrix}$$

$\hat{A}_{11}$ $\hat{A}_{12}$ $\hat{A}_{21}$ $\hat{A}_{22}$

Örnek devam

$$\underline{\hat{B}} = T^{-1}\underline{B}$$

$$\underline{\hat{B}} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{matrix} \\ \leftarrow \hat{B}_1 \\ \leftarrow \hat{B}_2 \end{matrix}$$

$$\left.\begin{matrix} q_1^T \\ q_2^T \end{matrix}\right\} \Longrightarrow \underline{\theta} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\bar{B} = \underline{\hat{B}}\theta^{-1} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ \cdots & \cdots \\ 0 & 1 \end{bmatrix}$$

$$\underline{u} = -\underline{K}\,\underline{x}$$

$$\underline{u} = -\underline{\hat{K}}\,\underline{\hat{x}} = -\underline{\hat{K}}T^{-1}\underline{x}$$

$$\boxed{\underline{K} = \underline{\hat{K}}T^{-1}}$$

İntegral Kontrollü Durum Geri Besleme

$$\underline{\hat{A}} - \underline{\hat{B}}\,\underline{\widehat{K}}$$

$$\underline{\hat{B}}\,\underline{\widehat{K}} = \underline{\bar{B}}\,\underline{\theta}\,\underline{\widehat{K}} = \underline{\bar{B}}\,\underline{\bar{K}}$$

$$\left|\lambda I - \left(\underline{\hat{A}} - \underline{\hat{B}}\,\underline{\widehat{K}}\right)\right| = \prod_{i=1}^{n}\left(\lambda - \hat{\lambda}_1\right)$$

$$\left|\lambda I - \left(\underline{\hat{A}} - \underline{\hat{B}}\,\underline{\widehat{K}}\right)\right| = \prod_{i=1}^{n}\left[\lambda - \left(\underline{\hat{A}}_{ii} - \underline{\bar{B}}_i\underline{\bar{K}}_{ii}\right)\right]$$

Örnek Devam

$$\underline{\hat{A}} - \underline{\hat{B}}\underline{\hat{K}} = \begin{bmatrix} 0 & 1 & 0 \\ -9 & -4 & 10 \\ -2 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{k}_{11} & \bar{k}_{12} & \bar{k}_{13} \\ \bar{k}_{21} & \bar{k}_{22} & \bar{k}_{23} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & \vdots & 0 \\ -9 - \bar{k}_{11} & -4 - \bar{k}_{12} & \vdots & 10 - \bar{k}_{13} \\ \cdots & \cdots & . & \cdots \\ -2 - k_{21} & \bar{k}_{22} & \vdots & 2 - \bar{k}_{23} \end{bmatrix}$$

$$\bar{k}_{13} = 10\ olsun$$

$$veya\ \bar{k}_{22} = 0\ \&\ \bar{k}_{21} = -2\ olsun$$

$$\underline{\hat{A}} - \underline{\hat{B}}\underline{\hat{K}} = \begin{bmatrix} 0 & 1 & \vdots & 0 \\ -9 - \bar{k}_{11} & -4 - \bar{k}_{12} & \vdots & 0 \\ \cdots & \cdots & . & \cdots \\ 0 & 0 & \vdots & 2 - \bar{k}_{23} \end{bmatrix}$$

Örnek Devam

$2 \times 2\ A_{11}$ bloğunu ele alırsak;

$$\left|\lambda I - \left(\underline{\hat{A}} - \underline{\hat{B}}\underline{\hat{K}}\right)\right| = \prod_{i=1}^{n}[\lambda - (\underline{\hat{A}}_{ii} - \underline{\bar{B}}_i \underline{\bar{K}}_{ii})]$$

$$\left|\lambda I - \begin{bmatrix} 0 & 1 \\ -9 - k_{11} & -4 - k_{12} \end{bmatrix}\right| = (\lambda - 2)(\lambda - 3)$$

$$\lambda^2 + \left(4 + \bar{k}_{12}\right)\lambda + \left(9 + \bar{k}_{11}\right) = \underbrace{\lambda^2 + 5\lambda + 6}_{\substack{-2\ ve -3\ ile \\ ilişkili}}$$

$$4 + \bar{k}_{12} = 5 \Rightarrow \bar{k}_{12} = 1$$

$$9 + \bar{k}_{11} = 6 \Rightarrow \bar{k}_{11} = -3$$

Örnek Devam

$$\underline{\hat{A}} - \underline{\hat{B}}\underline{\hat{K}} = \begin{bmatrix} 0 & 1 & \vdots & 0 \\ -9-\bar{k}_{11} & -4-\bar{k}_{12} & \vdots & 0 \\ \cdots & \cdots & \cdot & \cdots \\ 0 & 0 & \vdots & 2-\bar{k}_{23} \end{bmatrix}$$

$$2-\bar{k}_{23} = -1; istenen\ 3.\,özdeğer$$

$$\bar{k}_{23} = 3$$

$$\underline{\bar{K}} = \begin{bmatrix} -3 & 1 & 10 \\ -2 & 0 & 3 \end{bmatrix}$$

$$\underline{K} = \underline{\theta}^{-1}\underline{\bar{K}}T^{-1} = \underline{K} = \begin{bmatrix} -5-2\bar{k}_{21}+4\bar{k}_{22} & 1-2\bar{k}_{22} & -4-4\bar{k}_{22}+\bar{k}_{13} \\ \bar{k}_{21}-2\bar{k}_{22} & \bar{k}_{22} & 2\bar{k}_{22}+3 \end{bmatrix}$$